Government Science College-Gandhinagar

Title of course : Advanced Calculus, MAT-201

Problem Set-1 (Unit-1) by Yogita M. Parmar
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(i) Show that the limit does not exist in each case.

$(a)\ \lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$ $(e)\ \lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2}$

$(b)\ \lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2y^2+(x^2-y^2)^2}$ $(f)\ \lim_{(x,y)\to(0,0)} \frac{x^2+y}{y}$

$(c)\ \lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2}$ $(g)\ \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

$(d)\ \lim_{(x,y)\to(0,0)} \frac{x-y}{x+y}$ $(h)\ \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

(ii) Show that the limit exist in each case.

$(a)\ \lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$ $(b)\ \lim_{(x,y)\to(0,0)} \frac{x^4+y^4}{x^2+y^2}$ $(c)\ \lim_{(x,y)\to(0.0)} \frac{x^3-y^3}{x^2+y^2}$ $(d)\ \lim_{(x,y)\to(0,0)} \frac{\sqrt{x^2y^2+1}-1}{x^2+y^2}$

(iii) Show that the following functions are discontinuous at origin.

$(a)\ f(x,y) = \begin{cases} \frac{x^4+y^4}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases}$

$(b)\ f(x,y) = \begin{cases} \frac{x^3+y^3}{x-y}, x \neq y \\ 0, x = y \end{cases}$

$(c)\ f(x,y) = \begin{cases} \frac{1}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases}$

(iv) Discuss the continuity of the following function at origin.

$(a)\ f(x,y) = \begin{cases} x\sin\left(\frac{1}{y}\right) + y\sin\left(\frac{1}{x}\right), (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases}$

$(b)\ f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases}$

(v) Show that the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{2xy}{\sqrt{x^2+y^2}}, (x,y) \neq (0,0) \\ 0, \quad (x,y) = (0,0) \end{cases}$$

is continuous at origin.

(vi) Let a function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{2xy^2}{x^3+y^3}, (x,y) \neq (0,0) \\ 0, \quad (x,y) = (0,0) \end{cases}$$

Then show that $\lim_{x\to0}\left(\lim_{y\to0} f(x,y)\right)$ and $\lim_{y\to0}\left(\lim_{x\to0} f(x,y)\right)$ exists and are equal but the simultaneous limit $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

(vii) Examine the continuity of the following function at $(0,0)$

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, \quad (x,y) = (0,0) \end{cases}.$$

(viii) Examine the continuity of the function $f(x,y) = \sqrt{|xy|}$ at the origin.

(ix) Show that the function given by

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, \quad (x,y) = (0,0) \end{cases}$$

is continuous at $(0,0)$.

(x) Show that for the function

$$f(x,y) = \begin{cases} y\sin\left(\frac{1}{x}\right) + \frac{xy}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases}$$

$\lim_{x\to0}\lim_{y\to0} f(x,y)$ exists, but the other repeated limit and the double limit do not exist at the origin.